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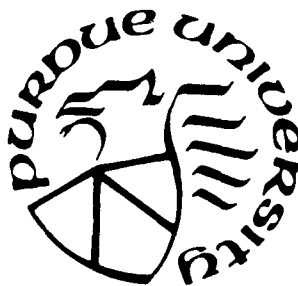
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with Symmetric Priors\*

by

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Technical Report # 89-13

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# EMPIRICAL BAYES ESTIMATION OF BINOMIAL PARAMETER WITH SYMMETRIC PRIORS

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## ABSTRACT

This paper deals with the problem of estimating the binomial parameter via the nonparametric empirical Bayes approach. This estimation problem has some surprising phenomenon that estimators which are asymptotically optimal in the usual empirical Bayes sense do not exist (Robbins (1956, 1964)). However, as pointed out by Liang (1984) and Gupta and Liang (1986), it is possible to construct asymptotically optimal empirical Bayes estimators if the unknown prior is symmetric about the point  $1/2$ . In this paper, assuming symmetric priors a monotone empirical Bayes estimator is constructed by using the isotonic regression method. This estimator is asymptotically optimal in the usual empirical Bayes sense. The corresponding rate of convergence is investigated and shown to be at least of order  $n^{-1/2}$ , where  $n$  is the number of past observations at hand.

Key Words and Phrases: Bayes estimator, empirical Bayes, asymptotically optimal, rate of convergence, isotonic regression, symmetric prior.

## 1. INTRODUCTION

Consider a sequence of Bernoulli process consisting of  $N$  trials. Let  $p$  denote the probability of success for each trial and  $Y$  stand for the number of successes among the total  $N$  trials. Then  $Y$  follows a binomial distribution with probability function  $f(y|p) = \binom{N}{y} p^y (1-p)^{N-y}$ ,  $y = 0, 1, \dots, N$ . Suppose that the parameter  $p$  is a realization of a random variable  $P$  having a prior distribution  $G$ . Thus, under the squared error loss, given  $Y = y$ , the Bayes estimator of  $p$  is the posterior mean of  $P$  given by

$$\varphi_G(y) = \frac{\int_0^1 p f(y|p) dG(p)}{\int_0^1 f(y|p) dG(p)} = \frac{w(y)}{h(y)} \quad (1.1)$$

where  $h(y) = \int_0^1 p^y (1-p)^{N-y} dG(p)$  and  $w(y) = \int_0^1 p^{y+1} (1-p)^{N-y} dG(p)$ . Also,  $f_G(y) = \binom{N}{y} h(y)$  is the marginal probability function of  $Y$ . The minimum Bayes risk is  $r(G) \equiv r(G, \varphi_G) = E[(\varphi_G(Y) - P)^2]$ .

When the prior distribution  $G$  is unknown, many authors, based on the past observations, treated this estimation problem via the empirical Bayes approach of Robbins (1956, 1964). For details, the reader is referred to Liang and Huang (1988), Vardeman (1978) and the related references. However, as pointed out by Robbins (1956, 1964), this estimation problem has some surprising phenomenon that estimators which are asymptotically optimal in the usual empirical Bayes sense do not exist. This is due to the fact that the function  $w(y)$  cannot be consistently estimated when the prior distribution  $G$  is completely unknown. To remedy this deficiency, Robbins (1956) suggested taking one more observations at each stage and proposed an estimator which is asymptotically optimal in a modified sense. Gupta and Liang (1989) treated this estimation problem through the parametric empirical Bayes approach assuming the prior to be a member of beta distribution family with unknown hyperparameters and then using the past observations to estimate

the unknown hyperparameters. Liang (1984) and Gupta and Liang (1986) have pointed out that if the unknown prior is symmetric about the point  $\frac{1}{2}$ , it is possible to construct asymptotically optimal empirical Bayes estimators for the binomial parameter  $p$ . However, no estimators were proposed.

In this paper, we deal with this estimation problem through the nonparametric empirical Bayes approach assuming symmetric priors. A monotone empirical Bayes estimator is constructed by using the isotonic regression method. This estimator is asymptotically optimal in the usual empirical Bayes sense. The corresponding rate of convergence is investigated and shown to be at least of order  $n^{-1}$  where  $n$  is the number of past observations at hand.

## 2. CONSTRUCTION OF EMPIRICAL BAYES ESTIMATORS

For each  $j = 1, 2, \dots$ , let  $(Y_j, P_j)$  be a pair of random variables where  $Y_j$  is observable but  $P_j$  is not. Conditional on  $P_j = p_j$ ,  $Y_j$  has a binomial probability function  $f(y|p_j) = \binom{N}{y} p_j^y (1 - p_j)^{N-y}$ ,  $y = 0, 1, \dots, N$ . It is assumed that  $P_j, j = 1, 2, \dots$ , are independently distributed with common unknown prior distribution  $G$ . Therefore,  $Y_j, j = 1, 2, \dots$ , are iid with marginal probability function  $f_G(y)$ . Let  $\underline{Y}_n = (Y_1, \dots, Y_n)$  denote the  $n$  past observations and  $Y_{n+1} = Y$  the current random observation. In the empirical Bayes estimation case, an estimation  $\varphi_n$  for the present problem is a function based on a sequence of past observations  $\underline{Y}_n$  and the present observation  $Y = y$ . We investigate this estimation problem under the following assumption.

**Assumption A:** The prior distribution  $G$  is symmetric about the point  $\frac{1}{2}$ , and  $N$  is an even number.

Under Assumption A, we have the following lemma which describes the relationship

between  $w(y)$  and  $h(y)$ .

**Lemma 2.1.** Under Assumption A, we have

- a)  $w\left(\frac{N}{2}\right) = \frac{1}{2}h\left(\frac{N}{2}\right)$ .
- b)  $w(x) = w(N - x - 1)$  for  $x = 0, 1, \dots, N - 1$ .
- c)  $w(x) + w(N - x) = h(x) = h(N - x)$ ,  $x = 0, 1, \dots, N$ .
- d)  $w(x) + w(x + 1) = h(x + 1)$ ,  $x = 0, 1, \dots, N - 1$ .
- e)  $\varphi_G(x) = 1 - \varphi_G(N - x)$ ,  $x = 0, 1, \dots, N$ , and
- f)  $w(x) = \sum_{i=0}^{x-\frac{N}{2}} h(x-i)(-1)^i - (-1)^{x-\frac{N}{2}} h\left(\frac{N}{2}\right)/2$ ,  $x = \frac{N}{2}, \dots, N$ .

**Proof:** Straight computation.

For each  $y = 0, 1, \dots, N$ , define

$$f_n(y) = f_n(N - y) = \begin{cases} \frac{1}{2^n} \sum_{j=1}^n I_{\{y, N-y\}}(Y_j) & \text{if } y \neq \frac{N}{2}, \\ \frac{1}{n} \sum_{j=1}^n I_{\{y\}}(Y_j) & \text{if } y = \frac{N}{2}. \end{cases}$$

$$h_n(y) = f_n(y) / \binom{N}{y}, \text{ and}$$

$$\begin{cases} w_n(y) = \sum_{i=0}^{y-\frac{N}{2}} h_n(y-i)(-1)^i - (-1)^{y-\frac{N}{2}} h_n\left(\frac{N}{2}\right)/2 & \text{if } N \geq y \geq \frac{N}{2}, \\ w_n(y) = h_n(y) - w_n(N - y) & \text{if } 0 \leq y \leq \frac{N}{2} - 1. \end{cases}$$

Both  $h_n(y)$  and  $w_n(y)$  are unbiased estimators of  $h(y)$  and  $w(y)$ , respectively,  $y = 0, 1, \dots, N$ . Thus, it is intuitive to use  $\frac{w_n(y)}{h_n(y)}$  as an estimator for  $\varphi_G(y) \equiv \frac{w(y)}{h(y)}$ . However, this naive estimator may have serious deficiencies. First,  $h_n(y)$  may be equal to zero and thus, the function  $\frac{w_n(y)}{h_n(y)}$  is not well defined. Second, it is possible that the value of  $\frac{w_n(y)}{h_n(y)}$  may be greater than 1 or less than 0, while  $0 \leq \varphi_G(y) \leq 1$  for all  $y = 0, 1, \dots, N$ . Hence, in the following, we seek a better estimator. The following lemma states the monotone

properties of the functions  $\varphi_G(y)$ ,  $h(y)$  and  $w(y)$ . These properties may suggest a way how to construct reasonable empirical Bayes estimators.

**Lemma 2.2.** a) For any prior distribution  $G$ ,  $\varphi_G(y)$  is an increasing function of  $y$ ,  $y = 0, 1, \dots, N$ .

b) Under Assumption A, both  $h(y)$  and  $w(y)$  are increasing in  $y$  for  $y = \frac{N}{2}, \dots, N$ .

Based on the monotone properties described in Lemma 2.2, we let  $\{\tilde{h}_n(x)\}_{x=\frac{N}{2}}^N$  be the isotonic regression of  $\{h_n(x)\}_{x=\frac{N}{2}}^N$  with equal weights and from Lemma 2.1, define  $\tilde{w}_n(x) = \sum_{i=0}^{x-\frac{N}{2}} \tilde{h}_n(x-i)(-1)^i - (-1)^{x-\frac{N}{2}} \tilde{h}_n(\frac{N}{2})/2$ , for  $\frac{N}{2} \leq x \leq N$ . Thus,  $\tilde{h}_n(x)$  is nondecreasing in  $x$  for  $\frac{N}{2} \leq x \leq N$ , and by this nondecreasing property,  $\tilde{w}_n(x) \geq 0$  for  $\frac{N}{2} \leq x \leq N$ . However,  $\tilde{w}_n(x)$  may still not possess the nondecreasing property. Thus, we let  $\{w_n^*(x)\}_{x=\frac{N}{2}}^N$  be the isotonic regression of  $\{\tilde{w}_n(x)\}_{x=\frac{N}{2}}^N$  with equal weights and from Lemma 2.1, define  $h_n^*(x) = w_n^*(x-1) + w_n^*(x)$  for  $\frac{N}{2} + 1 \leq x \leq N$  and  $h_n^*(\frac{N}{2}) = 2w_n^*(\frac{N}{2})$ . By the nondecreasing property of  $w_n^*(x)$ ,  $h_n^*(x)$  is nondecreasing in  $x$  for  $\frac{N}{2} \leq x \leq N$ . Now, for  $\frac{N}{2} \leq x \leq N$ , define

$$\varphi_n(x) = \begin{cases} \frac{w_n^*(x)}{h_n^*(x)} & \text{if } h_n^*(x) \neq 0, \\ \frac{1}{2} & \text{if } h_n^*(x) = 0. \end{cases}$$

Since  $\varphi_n(x)$  may be not a nondecreasing function of  $x$  for  $\frac{N}{2} \leq x \leq N$ , we consider the isotonic regression  $\{\varphi_n^*(x)\}_{x=\frac{N}{2}}^N$  of  $\{\varphi_n(x)\}_{x=\frac{N}{2}}^N$  with equal weights. Also, for  $0 \leq x \leq \frac{N}{2} - 1$ , define  $\varphi_n^*(x) = 1 - \varphi_n^*(N - x)$ . Now one can see that  $\varphi_n^*(x)$  is nondecreasing in  $x$  for  $x = 0, 1, \dots, N$ . We propose using  $\varphi_n^*(x)$  as an estimator of  $\varphi_G(x)$ ,  $x = 0, 1, \dots, N$ .

**Remark 2.1.** By the nondecreasing property of  $w_n^*(x)$  on  $x$ ,  $x = \frac{N}{2}, \dots, N$ ,  $\varphi_n(x) \geq \frac{1}{2}$  for all  $x \geq \frac{N}{2}$  and hence,  $\varphi_n^*(x) \geq \frac{1}{2}$  for  $x \geq \frac{N}{2}$ . Also,  $h_n^*(x) = 0$  iff  $w_n^*(x) = 0$  iff  $\tilde{w}_n(y) = 0$  for all  $\frac{N}{2} \leq y \leq x$  iff  $\tilde{h}_n(y) = 0$  for all  $\frac{N}{2} \leq y \leq x$  iff  $h_n(y) = 0$  for all  $y = N - x, \dots, x$ , where  $x \geq \frac{N}{2}$ .

### 3. ASYMPTOTIC OPTIMALITY

Let  $\psi_n(y)$  denote an empirical Bayes estimator based on the current observation  $y$  and the past data  $Y_n = (Y_1, \dots, Y_n)$ . Let  $r(G, \psi_n)$  denote the conditional Bayes risk (conditional on  $Y_n$ ) of the estimator  $\psi_n$  and  $Er(G, \psi_n)$  the associated overall Bayes risk where the expectation  $E$  is taken with respect to  $Y_n$ . Since  $r(G)$  is the minimum Bayes risk,  $r(G, \psi_n) - r(G) \geq 0$  and therefore  $Er(G, \psi_n) - r(G) \geq 0$ . The nonnegative difference  $Er(G, \psi_n) - r(G)$  is often used as a measure of the optimality of the empirical Bayes estimator  $\psi_n$ .

**Definition 3.1.** A sequence of empirical Bayes estimators  $\{\psi_n\}_{n=1}^{\infty}$  is said to be asymptotically optimal in  $E$  at least of order  $\beta_n$  relative to the prior distribution  $G$  if  $Er(G, \psi_n) - r(G) \leq O(\beta_n)$  where  $\{\beta_n\}_{n=1}^{\infty}$  is a sequence of positive numbers such that  $\lim_{n \rightarrow \infty} \beta_n = 0$ .

The usefulness of empirical Bayes estimators in practical applications clearly depend on the convergence rates for which the risks of the successive estimators approach the minimum Bayes risk. In the following, the performance of the proposed empirical Bayes estimators  $\{\varphi_n^*\}$  is evaluated on basis of the rates of convergence of the nonnegative difference  $Er(G, \varphi_n^*) - r(G)$ . Without loss of generality, we assume that  $G(0) < \frac{1}{2}$  to exclude the extreme case. In the following, all the computations are made under Assumption A.

**Lemma 3.1.** For each  $y = \frac{N}{2} + 1, \dots, N$ , suppose that  $h_n^*(y) > 0$ . Then for  $0 < t < \min(1 - \varphi_G(y), \varphi_G(y) - \frac{1}{2})$ ,

$$|\varphi_n^*(y) - \varphi_G(y)| > t \Rightarrow (N+2) \sum_{x=\frac{N}{2}}^N |h_n(x) - h(x)|^2 > [h(N/2)t]^2.$$

Proof:  $|\varphi_n^*(y) - \varphi_G(y)| > t \Rightarrow \varphi_n^*(y) - \varphi_G(y) > t$  or  $\varphi_n^*(y) - \varphi_G(y) < -t$ . By the definition of  $\varphi_n^*(y)$ ,  $w_n^*(x)$ ,  $\tilde{w}_n(x)$ ,  $\tilde{h}_n(x)$ , Lemma 2.1 and Theorem 2.1 of Barlow, et. al. (1972), we

have:

$$\varphi_n^*(y) - \varphi_G(y) > t$$

$$\Rightarrow \varphi_n(x) - \varphi_G(y) > t \quad \text{for some } N/2 + 1 \leq x \leq y.$$

$$\Rightarrow w_n^*(x)[1 - \varphi_G(y) - t] - w_n^*(x-1)[\varphi_G(y) + t] > 0 \text{ for some } N/2 + 1 \leq x \leq y$$

$$\Rightarrow [w_n^*(x) - w(x)][1 - \varphi_G(y) - t] - [w_n^*(x-1) - w(x-1)][\varphi_G(y) + t] > h(N/2)t$$

$$\text{for some } N/2 + 1 \leq x \leq y$$

$$\Rightarrow [w_n^*(x) - w(x)] > h(N/2)t \text{ or } [w_n^*(x-1) - w(x-1)] < -h(N/2)t$$

$$\text{for some } N/2 + 1 \leq x \leq y$$

$$\Rightarrow \sup_{\frac{N}{2} \leq x \leq N} |w_n^*(x) - w(x)| > h(N/2)t$$

$$\Rightarrow \sum_{x=\frac{N}{2}}^N |w_n^*(x) - w(x)|^2 > [h(N/2)t]^2$$

$$\Rightarrow \sum_{x=\frac{N}{2}}^N |\tilde{w}_n(x) - w(x)|^2 > [h(N/2)t]^2,$$

$$\text{since } \sum_{x=\frac{N}{2}}^N [w_n^*(x) - w(x)]^2 \leq \sum_{x=\frac{N}{2}}^N (\tilde{w}_n(x) - w(x))^2, \text{ see Theorem 2.1 of Barlow, et. al. (1972).}$$

Now, by the definition of  $\tilde{w}_n(x)$ , we have, for each  $x = \frac{N}{2}, \dots, N$ ,

$$\begin{aligned} & [\tilde{w}_n(x) - w(x)]^2 \\ &= \left[ \sum_{i=0}^{x-\frac{N}{2}} [\tilde{h}_n(x-i) - h(x-i)](-1)^i - (-1)^{x-\frac{N}{2}} [\tilde{h}_n(N/2) - h(N/2)]/2 \right]^2 \\ &\leq 2 \sum_{x=\frac{N}{2}}^N [\tilde{h}_n(x) - h(x)]^2 \\ &\leq 2 \sum_{x=\frac{N}{2}}^N [h_n(x) - h(x)]^2 \end{aligned}$$

where the last inequality is again from Theorem 2.1 of Barlow, et. al. (1972).

Based on the above discussions, we conclude that

$$\varphi_n^*(y) - \varphi_G(y) > t \Rightarrow (N+2) \sum_{x=\frac{N}{2}}^N [h_n(x) - h(x)]^2 > [h(N/2)t]^2. \quad (3.1)$$

Analogous to the preceding discussion, under the assumption that  $h_n^*(y) > 0$ , we can obtain:

$$\varphi_n^*(y) - \varphi_G(y) < -t \text{ and } h_n^*(y) > 0 \Rightarrow (N+2) \sum_{z=\frac{N}{2}}^N [h_n(z) - h(z)]^2 > [h(N/2)t]^2 \quad (3.2)$$

Therefore, (3.1) and (3.2) together lead to the result of the lemma.

**Remark 3.1.** Note that for each  $y = \frac{N}{2} + 1, \dots, N$ , as  $t > 1 - \varphi_G(y)$ ,  $\{\varphi_n^*(y) - \varphi_G(y) > t\} = \phi$ ; also, as  $t > \varphi_G(y) - \frac{1}{2}$ ,  $\{\varphi_n^*(y) - \varphi_G(y) < -t\} = \phi$ .

**Lemma 3.2.** For each  $y = \frac{N}{2} + 1, \dots, N$  and  $t > 0$ ,

$$P\{|\varphi_n^*(y) - \varphi_G(y)| > t \text{ and } h_n^*(y) > 0\} \leq \sum_{z=\frac{N}{2}}^N 2e^{-\frac{4nh^2(\frac{N}{2})(\frac{N}{z})^2 t^2}{(N+2)^2}}.$$

Proof: By Remark 3.1,  $P\{|\varphi_n^*(y) - \varphi_G(y)| > t, h_n^*(y) > 0\} = 0$  if  $t \geq \max(1 - \varphi_G(y), \varphi_G(y) - \frac{1}{2})$ . Thus, as  $0 < t < \max(1 - \varphi_G(y), \varphi_G(y) - \frac{1}{2})$ , from Lemma 3.1,

$$\begin{aligned} & P\{|\varphi_n^*(y) - \varphi_G(y)| > t, h_n^*(y) > 0\} \\ & \leq P\left\{\sum_{z=\frac{N}{2}}^N [h_n(z) - h(z)]^2 > \frac{[th(\frac{N}{2})]^2}{N+2}\right\} \\ & \leq \sum_{z=\frac{N}{2}}^N P\left\{|h_n(z) - h(z)| > \frac{\sqrt{2}th(\frac{N}{2})}{N+2}\right\} \\ & = \sum_{z=\frac{N}{2}}^N P\left\{|f_n(z) - f_G(z)| > \frac{\sqrt{2}th(\frac{N}{2})(\frac{N}{z})}{N+2}\right\} \\ & \leq \sum_{z=\frac{N}{2}}^N 2e^{-\frac{4nh^2(\frac{N}{2})(\frac{N}{z})^2 t^2}{(N+2)^2}} \end{aligned}$$

where the last inequality is obtained from Theorem 1 of Hoeffding (1963).

The following theorem is our main result.

**Theorem 3.1.** Let  $\{\varphi_n^*\}_{n=1}^\infty$  be the sequence of empirical Bayes estimators constructed in Section 2. Then, under Assumption A,

$$Er(G, \varphi_n^*) - r(G) \leq O(n^{-1}).$$

Proof: Straightforward computation leads to the following.

$$\begin{aligned}
0 &\leq Er(G, \varphi_n^*) - r(G) \\
&= \sum_{y=0}^N E[(\varphi_n^*(y) - \varphi_G(y))^2] f_G(y) \\
&= 2 \sum_{y=\frac{N}{2}+1}^N E[(\varphi_n^*(y) - \varphi_G(y))^2] f_G(y).
\end{aligned} \tag{3.3}$$

For each  $y = \frac{N}{2} + 1, \dots, N$ ,

$$\begin{aligned}
&E[(\varphi_n^*(y) - \varphi_G(y))^2] \\
&= \int_0^{\max(1-\varphi_G(y), \varphi_G(y)-\frac{1}{2})} 2tP\{|\varphi_n^*(y) - \varphi_G(y)| > t, h_n^*(y) > 0\} dt \\
&\quad + (\varphi_G(y) - 1/2)^2 P\{h_n^*(y) = 0\}.
\end{aligned}$$

Now, from Remark 2.1,

$$\begin{aligned}
P\{h_n^*(y) = 0\} &= P\{f_n(x) = 0 \text{ for all } x = N - y, \dots, y\} \\
&= [1 - F_G(y) + F_G(N - y - 1)]^n \\
&= \exp(-n \ln(1 - F_G(y) + F_G(N - y - 1)))^{-1} \\
&\leq O(n^{-1}).
\end{aligned} \tag{3.4}$$

where  $F_G(\cdot)$  is the marginal distribution function of  $Y$ . Also, from Lemma 3.2, and the fact that  $\max(1 - \varphi_G(y), \varphi_G(y) - \frac{1}{2}) < \frac{1}{2}$  for  $y \geq \frac{N}{2} + 1$ , we have

$$\begin{aligned}
&\int_0^{\max(1-\varphi_G(y), \varphi_G(y)-\frac{1}{2})} 2tP\{|\varphi_n^*(y) - \varphi_G(y)| > t, h_n^*(y) > 0\} dt \\
&\leq \int_0^{\frac{1}{2}} 4t \sum_{x=\frac{N}{2}}^N e^{-\frac{4nh^2(\frac{N}{2})(\frac{N}{x})^2 t^2}{(N+2)^2}} dt \\
&\leq \frac{1}{n} \sum_{x=\frac{N}{2}}^N \frac{(N+2)^2}{2h^2 (\frac{N}{2}) (\frac{N}{x})^2} \\
&= O(n^{-1}).
\end{aligned} \tag{3.5}$$

From (3.4) and (3.5), we conclude that for each  $y = \frac{N}{2} + 1, \dots, N$ ,

$$E[(\varphi_n^*(y) - \varphi_G(y))^2] \leq O(n^{-1}). \tag{3.6}$$

Since  $N$  is finite and fixed, (3.6) and (3.3) together complete the proof of the theorem.

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19. ABSTRACT (Continue on reverse if necessary and identify by block number) This paper deals with the problem of estimating the binomial parameter via the nonparametric empirical Bayes approach. This estimation problem has some surprising phenomenon that estimators which are asymptotically optimal in the usual empirical Bayes sense do not exist (Robbins (1956, 1964)). However, as pointed out by Liang (1984) and Gupta and Liang (1986), it is possible to construct asymptotically optimal empirical Bayes estimators if the unknown prior is symmetric about the point 1/2. In this paper, assuming symmetric priors a monotone empirical Bayes estimator is constructed by using the isotonic regression method. This estimator is asymptotically optimal in the usual empirical Bayes sense. The corresponding rate of convergence is investigated and shown to be at least of order $n^{-1}$ , where n is the number of past observations at hand.					
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